

UNIT II

PERFORMANCE OF SHORT, MEDIUM AND LONG TRANSMISSION LINES

CLASSIFICATION OF LINES - INTRODUCTION

The important considerations in the design and operation of a transmission line are the determination of voltage drop, line losses and efficiency of transmission. These values are greatly influenced by the line constants R , L and C of the transmission line. For instance the voltage drop in the line depends upon the values of above three line constants. Similarly, the resistance of transmission line conductors is the most important cause of power loss in the line and determines the transmission efficiency. In this chapter, we shall develop formulas by which we can calculate voltage regulation, line losses and efficiency of transmission lines. These formulas are important for two principal reasons. Firstly, they provide an opportunity to understand the effects of the parameters of the line on bus voltages and the flow of power. Secondly, they help in developing an overall understanding of what is occurring on electric power system.

CLASSIFICATION OF OVERHEAD TRANSMISSION LINES

A transmission line has three constants R , L and C distributed uniformly along the whole length of the line. The resistance and inductance form the series impedance. The capacitance existing between conductors for 1-phase line or from a conductor to neutral for a 3-phase line forms a shunt path throughout the length of the line. Therefore, capacitance effects introduce complications in transmission line calculations. Depending upon the manner in which capacitance is taken into account, the overhead transmission lines are classified as :

(i) Short transmission lines. When the length of an overhead transmission line is up to about 50 km and the line voltage is comparatively low (< 20 kV), it is usually considered as a short transmission line. Due to smaller length and lower voltage, the capacitance effects are small and hence can be neglected. Therefore, while studying the performance of a short transmission line, only resistance and inductance of the line are taken into account.

(ii) Medium transmission lines. When the length of an overhead transmission line is about 50-150 km and the line voltage is moderately high (>20 kV < 100 kV), it is considered as a medium transmission line. Due to sufficient length and voltage of the line, the capacitance effects are taken into account. For purposes of calculations, the distributed capacitance of the line is divided and lumped in the form of condensers shunted across the line at one or more points.

(iii) Long transmission lines. When the length of an overhead transmission line is more than 150 km and line voltage is very high (> 100 kV), it is considered as a long transmission line. For the treatment of such a line, the line constants are considered uniformly distributed over the whole length of the line and rigorous methods are employed for solution.

It may be emphasized here that exact solution of any transmission line must consider the fact that the constants of the line are not lumped but are distributed uniformly throughout the length of the line.

However, reasonable accuracy can be obtained by considering these constants as lumped for short and medium transmission lines.

Important Terms

While studying the performance of a transmission line, it is desirable to determine its voltage regulation and transmission efficiency. We shall explain these two terms in turn.

(i) Voltage regulation. When a transmission line is carrying current, there is a voltage drop in the line due to resistance and inductance of the line. The result is that receiving end voltage (V_R) of the line is generally less than the sending end voltage (V_S). This voltage drop ($V_S - V_R$) in the line is expressed as a percentage of receiving end voltage V and is called voltage regulation. The difference in voltage at the receiving end of a transmission line **between conditions of no load and full load is called **voltage regulation** and is expressed as a percentage of the receiving end voltage.

(ii) Transmission efficiency. The power obtained at the receiving end of a transmission line is generally less than the sending end power due to losses in the line resistance.

The ratio of receiving end power to the sending end power of a transmission line is known as the **transmission efficiency** of the line

PERFORMANCE OF SINGLE PHASE SHORT TRANSMISSION LINES

As stated earlier, the effects of line capacitance are neglected for a short transmission line. Therefore, while studying the performance of such a line, only resistance and inductance of the line are taken into account. The equivalent circuit of a single phase short transmission line is shown in Fig.

Here, the total line resistance and inductance are shown as concentrated or lumped instead of being distributed. The circuit is a simple a.c. series circuit.

Let

I = load current

R = loop resistance i.e., resistance of both conductors

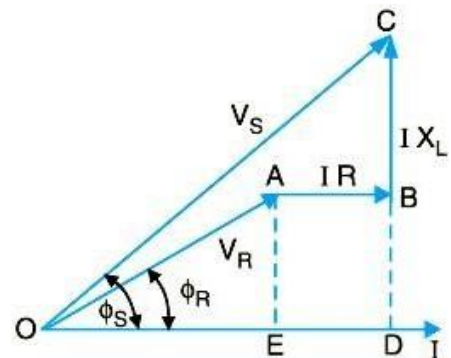
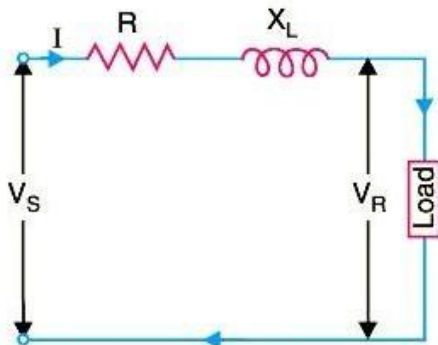
X_L = loop reactance

V_R = receiving end voltage

$\cos \phi_R$ = receiving end power factor (lagging)

V_S = sending end voltage

$\cos \phi_S$ = sending end power factor



The *phasor diagram of the line for lagging load power factor is shown in Fig. From the right angled triangle ODC, we get,

$$\begin{aligned}
 (OC)^2 &= (OD)^2 + (DC)^2 \\
 \text{or } V_S^2 &= (OE + ED)^2 + (DB + BC)^2 \\
 &= (V_R \cos \phi_R + IR)^2 + (V_R \sin \phi_R + IX_L)^2 \\
 \therefore V_S &= \sqrt{(V_R \cos \phi_R + IR)^2 + (V_R \sin \phi_R + IX_L)^2} \\
 \text{(i) \%age Voltage regulation} &= \frac{V_S - V_R}{V_R} \times 100 \\
 \text{(ii) Sending end } p.f., \cos \phi_S &= \frac{OD}{OC} = \frac{V_R \cos \phi_R + IR}{V_S} \\
 \text{(iii) Power delivered} &= V_R I_R \cos \phi_R \\
 \text{Line losses} &= I^2 R \\
 \text{Power sent out} &= V_R I_R \cos \phi_R + I^2 R \\
 \text{\%age Transmission efficiency} &= \frac{\text{Power delivered}}{\text{Power sent out}} \times 100 \\
 &= \frac{V_R I_R \cos \phi_R}{V_R I_R \cos \phi_R + I^2 R} \times 100
 \end{aligned}$$

An approximate expression for the sending end voltage V_S can be obtained as follows. Draw S perpendicular from B and C on OA produced as shown in Fig. Then OC is nearly equal to OF

$$\begin{aligned}
 OC &= OF = OA + AF = OA + AG + GF \\
 &= OA + AG + BH
 \end{aligned}$$

$$V_S = V_R + IR \cos \phi_R + I X_L \sin \phi_R$$

THREE-PHASE SHORT TRANSMISSION LINES

For reasons associated with economy, transmission of electric power is done by 3-phase system. This system may be regarded as consisting of three single phase units, each wire transmitting one-third of the total power. As a matter of convenience, we generally analyze 3-phase system by considering one phase only. Therefore, expression for regulation, efficiency etc. derived for a single phase line can also be applied to a 3-phase system. Since only one phase is considered, phase values of 3-phase system should be taken. Thus, V_S and V_R are the phase voltages, whereas R and X_L are the resistance S and inductive reactance per phase respectively.

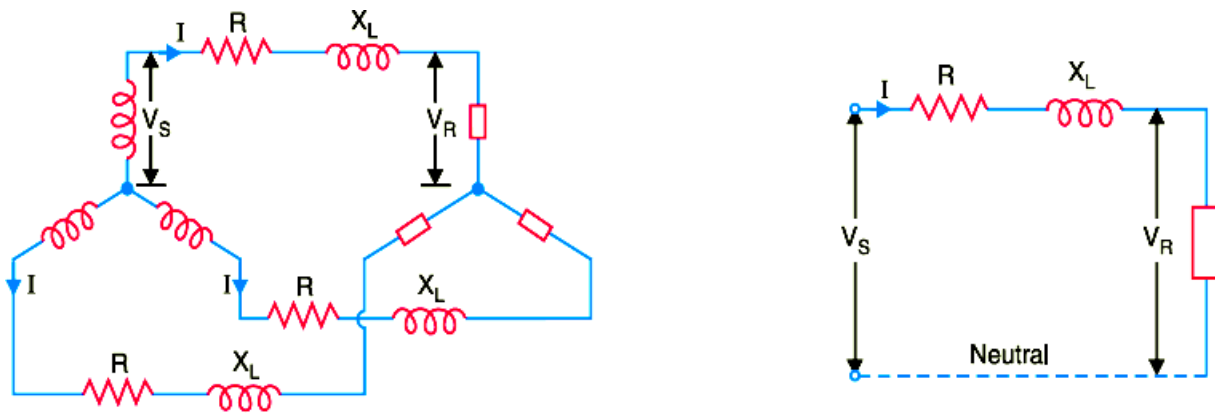


Fig (i) shows a Y-connected generator supplying a balanced Y-connected load through a transmission line. Each conductor has a resistance of $R \Omega$ and inductive reactance of $X \Omega$. Fig. (ii) shows one phase separately. The calculations can now be made in the same way as for a single phase line.

Effect of Load p.f. On Regulation and Efficiency

The regulation and efficiency of a transmission line depend to a considerable extent upon the power factor of the load.

1. Effect on regulation.

The expression for voltage regulation of a short transmission line is given by :

$$\% \text{age Voltage regulation} = \frac{IR \cos \phi_R + IX_L \sin \phi_R}{V_R} \times 100 \quad (\text{for lagging p.f.})$$

$$\% \text{age Voltage regulation} = \frac{IR \cos \phi_R - IX_L \sin \phi_R}{V_R} \times 100 \quad (\text{for leading p.f.})$$

The following conclusions can be drawn from the above expressions:

- (i) When the load p.f. is lagging or unity or such leading that $IR \cos \phi_R > IX_L \sin \phi_R$, then voltage regulation is positive *i.e.*, receiving end voltage V_R will be less than the sending end voltage V_S .
- (ii) For a given V_R and I , the voltage regulation of the line increases with the decrease in p.f. for lagging loads.
- (iii) When the load p.f. is leading to this extent that $IX_L \sin \phi_R > IR \cos \phi_R$, then voltage regulation is negative *i.e.* the receiving end voltage V_R is more than the sending end voltage V_S .
- (iv) For a given V_R and I , the voltage regulation of the line decreases with the decrease in p.f. for leading loads.

2. Effect on transmission efficiency.

The power delivered to the load depends upon the power factor.

$$P = V_R * I \cos \phi_R \quad (\text{For 1-phase line})$$

$$I = \frac{P}{V_R \cos \phi_R}$$

$$P = 3 V_R I \cos \phi_R \quad (\text{For 3-phase line})$$

$$I = \frac{P}{3V_R \cos \phi_R}$$

It is clear that in each case, for a given amount of power to be transmitted (P) and receiving end voltage Power Factor Meter (V_R), the load current I is inversely proportional to the load p.f. cos ϕ_R . Consequently, with the decrease in load p.f., the load current and hence the line losses are increased. This leads to the conclusion that transmission efficiency of a line decreases with the decrease in load Power Factor Regulator p.f. and vice-versa,

ABCD PARAMETERS

A major section of power system engineering deals in the transmission of electrical power from one particular place (eg. Generating station) to another like substations or distribution units with maximum efficiency. So its of substantial importance for power system engineers to be thorough with its mathematical modeling. Thus the entire transmission system can be simplified to a **two port network** for the sake of easier calculations.

The circuit of a 2 port network is shown in the diagram below. As the name suggests, a 2 port network consists of an input port PQ and an output port RS. Each port has 2 terminals to

Connect itself to the external circuit. Thus it is essentially a 2 port or a 4 terminal circuit, having

Supply end voltage = V_S and Supply end current = I_S given to the input port P Q.

And there is the Receiving end Voltage = V_R and Receiving end current = I_R

Given to the output port R S. As shown in the diagram below.

Now the **ABCD parameters** or the transmission line parameters provide the link between the supply and receiving end voltages and currents, considering the circuit elements to be linear in nature.

Thus the relation between the sending and receiving end specifications are given using

ABCD parameters by the equations below.

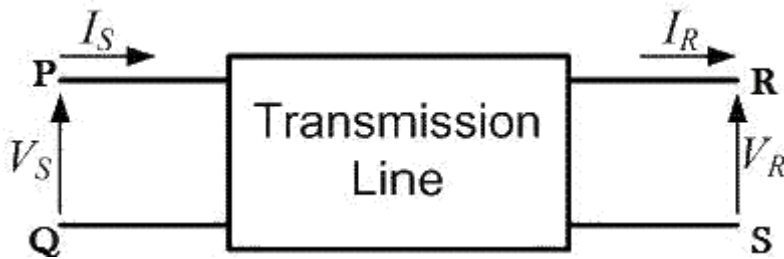
$$V_S = A V_R + B I_R \quad \text{—————(1)}$$

$$I_S = C V_R + D I_R \quad \text{—————(2)}$$

Now in order to determine the ABCD parameters of transmission line let us impose the required circuit conditions in different cases.

ABCD parameters, when receiving end is open circuited

The receiving end is open circuited meaning receiving end current $I_R = 0$. Applying this condition to



equation (1) we get.

$$V_S = A V_R + B \cdot 0 \Rightarrow V_S = A V_R + 0$$

$$A = \left. \frac{V_S}{V_R} \right|_{I_R = 0}$$

Thus it implies that on applying open circuit condition to ABCD parameters, we get parameter A as the ratio of sending end voltage to the open circuit receiving end voltage. Since dimension wise A is a ratio of voltage to voltage, A is a dimensionless parameter.

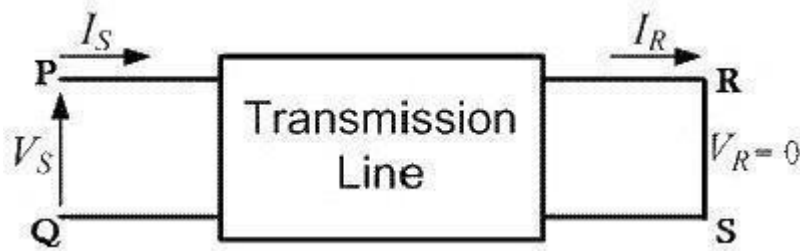
Applying the same open circuit condition i.e $I_R = 0$ to equation (2)

$$I_S = C V_R + D \cdot 0 \Rightarrow I_S = C V_R + 0$$

$$C = \left. \frac{I_S}{V_R} \right|_{I_R = 0}$$

Thus it implies that on applying open circuit condition to ABCD parameters of transmission line, we get parameter C as the ratio of sending end current to the open circuit receiving end voltage. Since dimension wise C is a ratio of current to voltage, its unit is mho.

Thus C is the open circuit conductance and is given by $C = I_S/V_R$ mho.

ABCD parameters when receiving end is short circuited

Receiving end is short circuited meaning receiving end voltage $V_R = 0$ Applying this condition to equation (1) we get

$$V_S = A \cdot 0 + B I_R \Rightarrow V_S = 0 + B I_R$$

$$B = \left. \frac{V_S}{I_R} \right|_{V_R = 0}$$

Thus it implies that on applying short circuit condition to ABCD parameters, we get parameter B as the ratio of sending end voltage to the short circuit receiving end current. Since dimension wise B is a ratio of voltage to current, its unit is Ω . Thus B is the short circuit resistance and is

given by

$$B = V_S / I_R \Omega.$$

Applying the same short circuit condition i.e $V_R = 0$ to equation (2) we get

$$I_S = C \cdot 0 + D I_R \Rightarrow I_S = 0 + D I_R$$

$$D = \left. \frac{I_S}{I_R} \right|_{V_R = 0}$$

Thus it implies that on applying short circuit condition to ABCD parameters, we get parameter D as the ratio of sending end current to the short circuit receiving end current. Since dimension wise D is a ratio of current to current, it's a dimensionless parameter. $\therefore$ the ABCD parameters of transmission line can be tabulated as:-

Parameter Specification

Unit A = V_S / V_R Voltage ratio

Unitless **B** = V_S / I_R

Short circuit resistance Ω

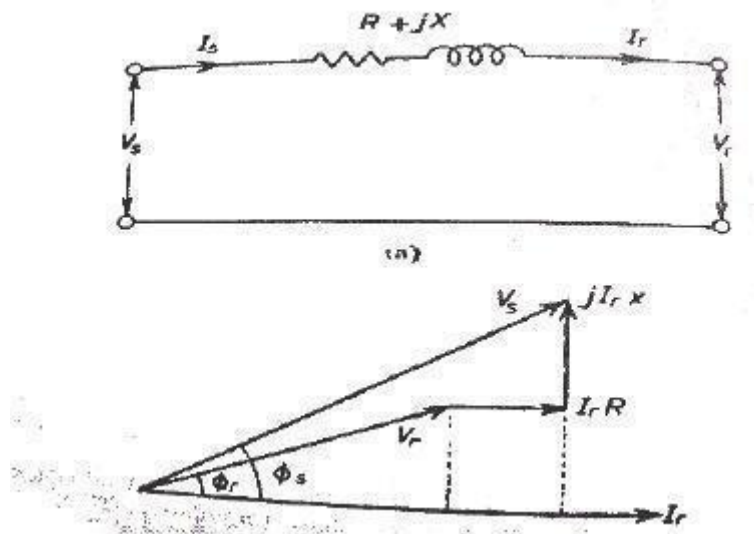
C = I_S / V_R Open circuit conductance mho

D = I_S / I_R Current ratio Unitless

SHORT TRANSMISSION LINE

The transmission lines which have length less than 80 km are generally referred as short transmission lines. For short length, the shunt capacitance of this type of line is neglected and other parameters like resistance and inductance of these short lines are lumped, hence the equivalent circuit is represented as given below,

Let's draw the vector diagram for this equivalent circuit, taking receiving end current I_r as reference. The sending end and receiving end voltages make angle with that reference receiving end current, of ϕ_s and ϕ_r , respectively.



As the shunt capacitance of the line is neglected, hence sending end current and receiving end current is same, i.e.

$$I_s = I_r.$$

Now if we observe the vector diagram carefully, we will get, V_s is approximately equal to $V_r + I_r R \cos \phi_r + I_r X \sin \phi_r$ that means,

$V_s \cong V_r + I_r R \cos \phi_r + I_r X \sin \phi_r$ as it is assumed that $\phi_s \cong \phi_r$

As there is no capacitance, during no load condition the current through the line is considered as zero, hence at no load condition, receiving end voltage is the same as sending end voltage. As per definition of voltage regulation,

$$\% \text{ regulation} = \frac{V_s - V_r}{V_r} \times 100 \%$$

$$= \frac{I_r R \cos \phi_r + I_r X \sin \phi_r}{V_r} \times 100 \%$$

$$\text{per unit regulation} = \frac{I_r R}{V_r} \cos \phi_r + \frac{I_r X}{V_r} \sin \phi_r = v_r \cos \phi_r + v_x \sin \phi_r$$

$$A = \left. \frac{V_s}{V_r} \right|_{I_r = 0}$$

Here, v_r and v_x are the per unit resistance and reactance of the short transmission line.

Any electrical network generally has two input terminals and two output terminals. If we consider any complex electrical network in a black box, it will have two input terminals and output terminals. This network is called two – port network. Two port model of a network simplifies the network solving technique. Mathematically a two port network can be solved by 2 by 2 matrixes.

A transmission as it is also an electrical network; line can be represented as two port network. Hence two port network of transmission line can be represented as 2 by 2 matrixes. Here the concept of ABCD parameters comes. Voltage and currents of the network can be represented as ,

$$V_s = AV_r + BI_r \dots \quad (1)$$

$$I_s = CV_r + DI_r \dots \quad (2)$$

Where A, B, C and D are different constant of the network. If we put $I_r = 0$ at equation (1), we get

Hence, A is the voltage impressed at the sending end per volt at the receiving end when receiving end is open. It is dimension less.

If we put $V_r = 0$ at equation (1), we get

$$B = \left. \frac{V_s}{I_r} \right|_{V_r = 0}$$

That indicates it is impedance of the transmission line when the receiving terminals are short circuited. This parameter is referred as transfer impedance.

$$C = \left. \frac{I_s}{V_r} \right|_{I_r = 0}$$

C is the current in amperes into the sending end per volt on open circuited receiving end. It has the dimension of admittance.

$$D = \left. \frac{I_s}{I_r} \right|_{V_r = 0}$$

D is the current in amperes into the sending end per amp on short circuited receiving end. It is dimensionless.

Now from equivalent circuit, it is found that,

$V_s = V_r + I_r Z$ and $I_s = I_r$ Comparing these equations with equation 1 and 2 we get,

$A = 1$, $B = Z$, $C = 0$ and $D = 1$. As we know that the constant A, B, C and D are related for passive network as

Here, $A = 1$, $B = Z$, $C = 0$ and $D = 1$

$$AD - BC = 1.$$

$$\Rightarrow 1.1 - Z.0 = 1$$

So the values calculated are correct for short transmission line. From above equation (1),

$$V_s = AV_r + BI_r$$

When $I_r = 0$ that means receiving end terminals is open circuited and then from the equation 1, we get receiving end voltage at no load

$$V_{r'} = \frac{V_s}{A}$$

and as per definition of voltage regulation,

$$\% \text{ voltage regulation} = \frac{V_s / A - V_r}{V_r} \times 100 \%$$

Efficiency of Short Transmission Line

The efficiency of short line as simple as efficiency equation of any other electrical equipment, that means

$$\begin{aligned} \% \text{ efficiency } (\mu) &= \frac{\text{Power received at receiving end}}{\text{Power delivered at sending end}} \times 100 \% \\ \% \mu &= \frac{\text{Power received at receiving end}}{\text{Power received at receiving end} + 3I_r^2 R} \times 100 \% \end{aligned}$$

MEDIUM TRANSMISSION LINES

In short transmission line calculations, the effects of the line capacitance are neglected because such lines have smaller lengths and transmit power at relatively low voltages (< 20 kV). However, as the length and voltage of the line increase, the capacitance gradually becomes of greater importance.

Since medium transmission lines have sufficient length (50-150 km) and usually operate at voltages greater than 20 kV, the effects of capacitance cannot be neglected. Therefore, in order to obtain reasonable accuracy in medium transmission line calculations, the line capacitance must be taken into consideration.

The capacitance is uniformly distributed over the entire length of the line. However, in order to make the calculations simple, the line capacitance is assumed to be lumped or concentrated in the form of capacitors shunted across the line at one or more points. Such a treatment of localising the

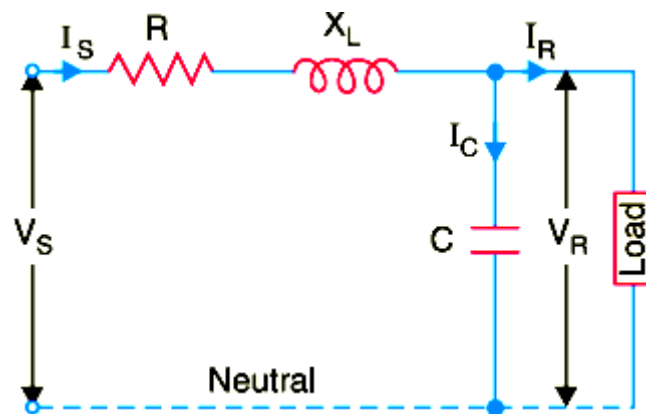
line capacitance gives reasonably accurate results. The most commonly used methods (known as localised capacitance methods) for the solution of medium transmission lines are:

- (i) End condenser method
- (ii) Nominal T method
- (iii) Nominal π method.

Although the above methods are used for obtaining the performance calculations of medium lines, they can also be used for short lines if their line capacitance is given in a particular problem.

i) End Condenser Method

In this method, the capacitance of the line is lumped or concentrated at the receiving or load end as shown in Fig. This method of localising the line capacitance at the load end overestimates the effects of capacitance. In Fig, one phase of the 3-phase transmission line is shown as it is more convenient to work in phase instead of line-to-line values.



Let

- I_R = load current per phase
- R = resistance per phase
- X_L = inductive reactance per phase
- C = capacitance per phase
- $\cos \phi_R$ = receiving end power factor (lagging)
- V_s = sending end voltage per phase

The *phasor diagram for the circuit is shown in Fig Taking the receiving end voltage V_R as the reference phasor,

we have, $\vec{V}_R = V_R + j 0$

Load current, $\vec{I}_R = I_R (\cos \phi_R - j \sin \phi_R)$

Capacitive current, $\vec{I}_C = j \vec{V}_R \omega C = j 2 \pi f C \vec{V}_R$

The sending end current I_s is the phasor sum of load current I_R and capacitive current I_C i.e.

$$\begin{aligned}
 \vec{I}_S &= \vec{I}_R + \vec{I}_C \\
 &= I_R (\cos \phi_R - j \sin \phi_R) + j 2 \pi f C V_R \\
 &= I_R \cos \phi_R + j (-I_R \sin \phi_R + 2 \pi f C V_R) \\
 &= \vec{I}_S \vec{Z} = \vec{I}_S (R + j X_L)
 \end{aligned}$$

$$\vec{V}_S = \vec{V}_R + \vec{I}_S \vec{Z} = \vec{V}_R + \vec{I}_S (R + j X_L)$$

Thus, the magnitude of sending end voltage V_S can be calculated.

$$\% \text{ Voltage regulation} = \frac{V_S - V_R}{V_R} \times 100$$

$$\begin{aligned}
 \% \text{ Voltage transmission efficiency} &= \frac{\text{Power delivered / phase}}{\text{Power delivered / phase} + \text{losses / phase}} \times 100 \\
 &= \frac{V_R I_R \cos \phi_R}{V_R I_R \cos \phi_R + I_S^2 R} \times 100
 \end{aligned}$$

Limitations

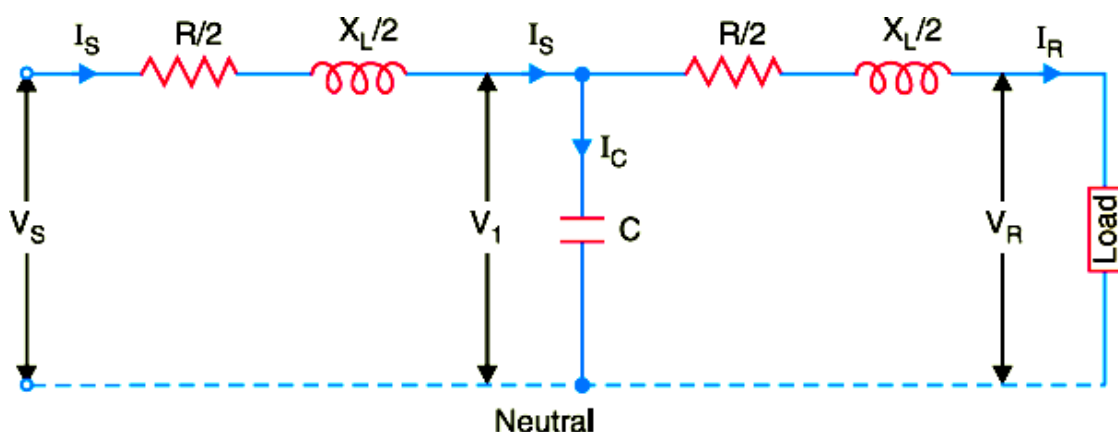
Although end condenser method for the solution of medium lines is simple to work out calculations, yet it has the following drawbacks:

(i) There is a considerable error (about 10%) in calculations because the distributed capacitance has been assumed to be lumped or concentrated.

(ii) This method overestimates the effects of line capacitance.

ii) Nominal T Method

In this method, the whole line capacitance is assumed to be concentrated at the middle point of the line and half the line resistance and reactance are lumped on its either side as shown in Fig. Therefore, in this arrangement, full charging current flows over half the line. In Fig. one phase of 3-phase transmission line is shown as it is advantageous to work in phase instead of line-to-line values.



Let

I_R = load current per phase; R =
resistance per phase

X_L = inductive reactance per phase;

C = capacitance per phase

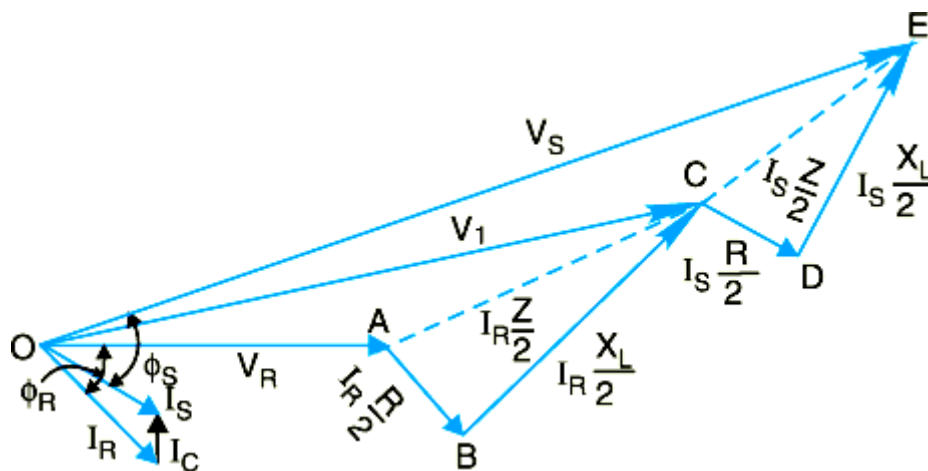
$\cos \phi_R$ = receiving end power factor (lagging) ; V_S =
sending end voltage/phase

V_1 = voltage across capacitor C

The *phasor diagram for the circuit is shown in Fig. Taking the receiving end voltage V_R as the reference phasor, we have,

$$\text{Receiving end voltage, } \vec{V}_R = V_R + j0$$

$$\text{Load current, } \vec{I}_R = I_R (\cos \phi_R - j \sin \phi_R)$$



$$\begin{aligned} \text{Voltage across } C, \quad \vec{V}_1 &= \vec{V}_R + \vec{I}_R \vec{Z} / 2 \\ &= V_R + I_R (\cos \phi_R - j \sin \phi_R) \left(\frac{R}{2} + j \frac{X_L}{2} \right) \end{aligned}$$

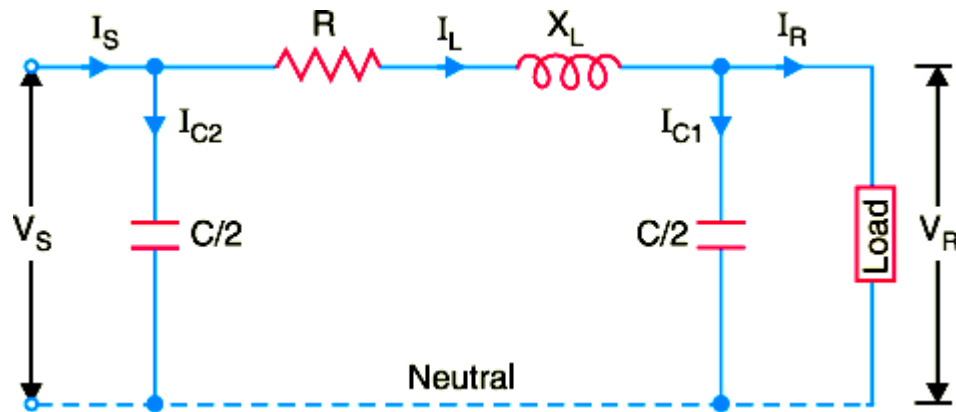
$$\text{Capacitive current, } \vec{I}_C = j \omega C \vec{V}_1 = j 2\pi f C \vec{V}_1$$

$$\text{Sending end current, } \vec{I}_S = \vec{I}_R + \vec{I}_C$$

$$\text{Sending end voltage, } \vec{V}_S = \vec{V}_1 + \vec{I}_S \frac{\vec{Z}}{2} = \vec{V}_1 + \vec{I}_S \left(\frac{R}{2} + j \frac{X_L}{2} \right)$$

iii) Nominal π Method

In this method, capacitance of each conductor (i.e., line to neutral) is divided into two halves; one half being lumped at the sending end and the other half at the receiving end as shown in Fig. It is obvious that capacitance at the sending end has no effect on the line drop. However, its charging current must be added to line current in order to obtain the total sending end current.



Let

I_R = load current per phase R = resistance per phase

X_L = inductive reactance per phase C = capacitance per phase

$\cos \phi_R$ = receiving end power factor (lagging) V_S = sending end voltage per phase

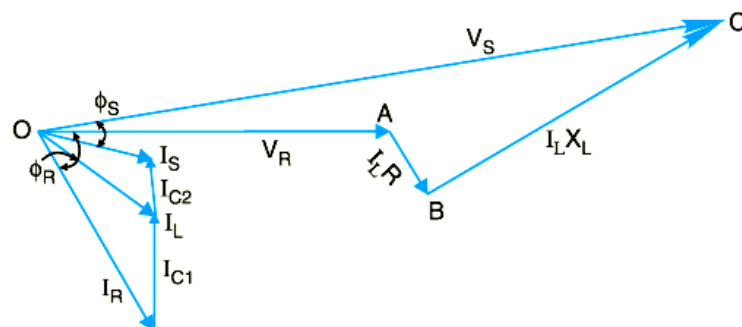
The *phasor diagram for the circuit is shown in Fig. Taking the receiving end voltage as the reference phasor, we have,

$$\vec{V}_R = V_R + j0$$

$$\text{Load current, } \vec{I}_R = I_R (\cos \phi_R - j \sin \phi_R)$$

Charging current at load end is

$$\vec{I}_{C1} = j \omega (C/2) \vec{V}_R = j \pi f C \vec{V}_R$$



Line current, $\vec{I}_L = \vec{I}_R + \vec{I}_{C1}$

Sending end voltage, $\vec{V}_S = \vec{V}_R + \vec{I}_L \vec{Z} = \vec{V}_R + \vec{I}_L (R + jX_L)$

Charging current at the sending end is

$$\vec{I}_{C2} = j \omega (C/2) \vec{V}_S = j \pi f C \vec{V}_S$$

$\therefore$ Sending end current, $\vec{I}_S = \vec{I}_L + \vec{I}_{C2}$

MEDIUM TRANSMISSION LINE

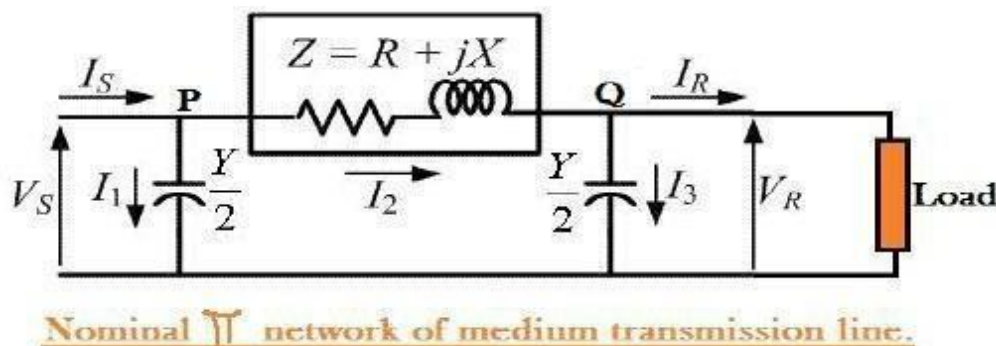
The transmission line having its effective length more than 80 km but less than 250 km is generally referred to as a **medium transmission line**. Due to the line length being considerably high, admittance Y of the network does play a role in calculating the effective circuit parameters, unlike in the case of short transmission lines. For this reason the modelling of a **medium length transmission line** is done using lumped shunt admittance along with the lumped impedance in series to the circuit. These lumped parameters of a medium length transmission line can be represented using two different models, namely.

- 1) Nominal Π representation.
- 2) Nominal T representation.

Let's now go into the detailed discussion of these above mentioned models.

Nominal Π representation of a medium transmission line

In case of a nominal Π representation, the lumped series impedance is placed at the middle of the circuit where as the shunt admittances are at the ends. As we can see from the diagram of the Π network below, the total lumped shunt admittance is divided into 2 equal halves, and each half with value $Y/2$ is placed at both the sending and the receiving end while the entire circuit impedance is between the two. The shape of the circuit so formed resembles that of a symbol Π , and for this reason it is known as the nominal Π representation of a medium transmission line. It is mainly used for determining the general circuit parameters and performing load flow analysis.



As we can see here, V_S and V_R is the supply and receiving end voltages respectively, and I_S is the current flowing through the supply end.

I_R is the current flowing through the receiving end of the circuit.

I_1 and I_3 are the values of currents flowing through the admittances. And I_2 is the current through the impedance Z .

Now applying KCL, at node P, we get. $I_S = I_1 + I_2$ —————(1)

Similarly applying KCL, to node Q. $I_2 = I_3 + I_R$ —————(2)

Now substituting equation (2) to equation(1)

$$I_S = I_1 + I_3 + I_R$$

$$= \frac{Y}{2} V_S + \frac{Y}{2} V_R + I_R \text{ —————(3)}$$

Now by applying KVL to the circuit, $V_S = V_R + Z I_2$

$$\begin{aligned} &= V_R + Z \left(V_R \frac{Y}{2} + I_R \right) \\ &= \left(Z \frac{Y}{2} + 1 \right) V_R + Z I_R \text{ —————(4)} \end{aligned}$$

Now substituting equation (4) to equation (3), we get.

$$\begin{aligned} I_S &= \frac{Y}{2} \left[\left(\frac{Y}{2} Z + 1 \right) V_R + Z I_R \right] + \frac{Y}{2} V_R + I_R \\ &= Y \left(\frac{Y}{4} Z + 1 \right) V_R + \left(\frac{Y}{2} Z + 1 \right) I_R \text{ —————(5)} \end{aligned}$$

Comparing equation (4) and (5) with the standard ABCD parameter equations

$$V_S = A V_R + B I_R$$

$$I_S = C V_R + D I_R$$

We derive the parameters of a medium transmission line as:

$$A = \left(\frac{Y}{2} Z + 1 \right)$$

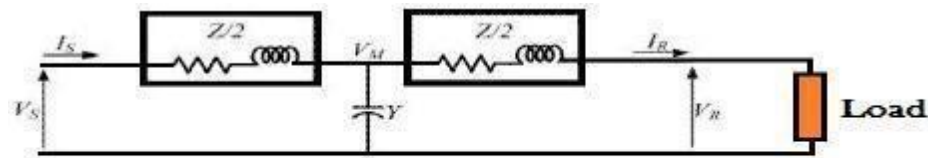
$$B = Z \Omega$$

$$C = Y \left(\frac{Y}{4} Z + 1 \right)$$

$$D = \left(\frac{Y}{2} Z + 1 \right)$$

NOMINAL T REPRESENTATION OF A MEDIUM TRANSMISSION LINE

In the **nominal T** model of a medium transmission line the lumped shunt admittance is placed in the middle, while the net series impedance is divided into two equal halves and placed on either side of the shunt admittance. The circuit so formed resembles the symbol of a capital **T**, and hence is known as the nominal T network of a medium length transmission line and is shown in the diagram below.



Nominal T representation of a medium transmission line.

Here also V_S and V_R is the supply and receiving end voltages respectively, and I_S is the current flowing through the supply end. I_R is the current flowing through the receiving end of the circuit. Let M be a node at the midpoint of the circuit, and the drop at M , be given by V_M . Applying KVL to the above network we get

$$\frac{V_S - V_M}{Z/2} = Y V_M + \frac{V_M - V_R}{Z/2}$$

$$\text{Or } V_M = \frac{2(V_S + V_R)}{YZ + 4} \quad (6)$$

And the receiving end current

$$\text{Or } I_R = \frac{2(V_M - V_R)}{Z/2} \quad (7)$$

Now substituting V_M from equation (6) to (7) we get,

$$\text{Or } I_R = \frac{[(2V_S + V_R) / YZ + 4] - V_R}{Z/2}$$

Rearranging the above equation:

$$V_S = \left(\frac{Y}{2}Z + 1\right)V_R + Z\left(\frac{Y}{4}Z + 1\right)I_R \quad (8)$$

Now the sending end current is

$$I_S = Y V_M + I_R \quad (9)$$

Substituting the value of V_M to equation (9) we get,

$$\text{Or } I_S = Y V_R + \left(\frac{Y}{2}Z + 1\right)I_R \quad (10)$$

Again comparing Comparing equation (8) and (10) with the standard ABCD parameter equations

$$V_S = A V_R + B I_R$$

$$I_S = C V_R + D I_R$$

The parameters of the T network of a medium transmission line are

$$A = \left(\frac{Y}{2}Z + 1\right)$$

$$B = Z\left(\frac{Y}{4}Z + 1\right) \Omega$$

$$C = Y \text{ mho}$$

$$D = \left(\frac{Y}{2}Z + 1\right)$$

LONG TRANSMISSION LINES

It is well known that line constants of the transmission line are uniformly distributed over the entire length of the line. However, reasonable accuracy can be obtained in line calculations for short and medium lines by considering these constants as lumped. If such an assumption of lumped constants is applied to long transmission lines (having length excess of about 150 km), it is found that serious errors are introduced in the performance calculations. Therefore, in order to obtain fair degree of accuracy in the performance calculations of long lines, the line constants are considered as uniformly distributed throughout the length of the line. Rigorous mathematical treatment is required for the solution of such lines. Fig shows the equivalent circuit of a 3-phase long transmission line on a phase-neutral basis. The whole line length is divided into n sections, each section having line constants $1/n$ th of those for the whole line. The following points may be noted :

- (i) The line constants are uniformly distributed over the entire length of line as is actually the case.
- (ii) The resistance and inductive reactance are the series elements.
- (ii) The leakage susceptance (B) and leakage conductance (G) are shunt elements.
- (iii) The leakage susceptance is due to the fact that capacitance exists between line and neutral. The leakage conductance takes into account the energy losses occurring through leakage over the

$$= \sqrt{G^2 + B^2} .$$

insulators or due to corona effect between conductors. Admittance

- (iv) The leakage current through shunt admittance is maximum at the sending end of the line and decreases continuously as the receiving end of the circuit is approached at which point its value is zero.

ANALYSIS OF LONG TRANSMISSION LINE (RIGOROUS METHOD)

Fig. shows one phase and neutral connection of a 3-phase line with impedance and shunt admittance of the line uniformly distributed.

Consider a small element in the line of length dx situated at a distance x from the receiving end.

Let

z = series impedance of the line per unit length

y = shunt admittance of the line per unit length

V = voltage at the end of element towards receiving end

$V + dV$ = voltage at the end of element towards sending end

$I + dI$ = current entering the element

I = current leaving the element dx

Then for the small element dx ,

$z dx$ = series impedance y

dx = shunt admittance

Obviously, $dV = I z dx$

$$\frac{dV}{dx} = I z$$

Now, the current entering the element is $I + dI$ whereas the current leaving the element is I . The difference in the currents flows through shunt admittance of the element i.e.,

dI = Current through shunt admittance of element = $V y dx$

$$\text{or} \quad \frac{dI}{dx} = V y \quad \dots(ii)$$

Differentiating eq. (i) w.r.t. x , we get,

$$\frac{d^2 V}{dx^2} = z \frac{dI}{dx} = z (V y) \quad \left[\because \frac{dI}{dx} = V y \text{ from exp. (ii)} \right]$$

$$\text{or} \quad \frac{d^2 V}{dx^2} = y z V \quad \dots (iii)$$

The solution of this differential equation is

$$V = k_1 \cosh(x \sqrt{y z}) + k_2 \sinh(x \sqrt{y z}) \quad \dots(iv)$$

$$\text{or} \quad \frac{dI}{dx} = V y \quad \dots(ii)$$

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The solution of this differential equation is

$$V = k_1 \cosh(x \sqrt{y z}) + k_2 \sinh(x \sqrt{y z}) \quad \dots(iv)$$

Equations (iv) and (v) give the expressions for V and I in the form of unknown constants k_1 and K_2 . The values of k_1 and k_2 can be found by applying end conditions as under

At $x = 0$, $V = V_R$ and $I = I_R$

Putting these values in eq. (iv), we have,

$$V_R = k_1 \cosh 0 + k_2 \sinh 0 = k_1 + 0$$

$$\therefore V_R = k_1$$

Similarly, putting $x = 0$, $V = V_R$ and $I = I_R$ in eq. (v), we have,

$$I_R = \sqrt{\frac{y}{z}} [k_1 \sinh 0 + k_2 \cosh 0] = \sqrt{\frac{y}{z}} [0 + k_2]$$

$$\therefore k_2 = \sqrt{\frac{z}{y}} I_R$$

Substituting the values of k_1 and k_2 in eqs. (iv) and (v), we get,

$$V = V_R \cosh (x\sqrt{yz}) + \sqrt{\frac{z}{y}} I_R \sinh (x\sqrt{yz})$$

and

$$I = \sqrt{\frac{y}{z}} V_R \sinh (x\sqrt{yz}) + I_R \cosh (x\sqrt{yz})$$

The sending end voltage (V_S) and sending end current (I_S) are obtained by putting $x = l$ in the above equations *i.e.*,

$$V_S = V_R \cosh (l \sqrt{y z}) + \sqrt{\frac{z}{y}} I_R \sinh (l \sqrt{y z})$$

$$I_S = \sqrt{\frac{y}{z}} V_R \sinh (l \sqrt{y z}) + I_R \cosh (l \sqrt{y z})$$

Now,

$$l \sqrt{y z} = \sqrt{l y \cdot l z} = \sqrt{Y Z}$$

and

$$\sqrt{\frac{y}{z}} = \sqrt{\frac{y l}{z l}} = \sqrt{\frac{Y}{Z}}$$

where

Y = total shunt admittance of the line

Z = total series impedance of the line

Therefore, expressions for V_S and I_S become :

$$V_S = V_R \cosh \sqrt{Y Z} + I_R \sqrt{\frac{Z}{Y}} \sinh \sqrt{Y Z}$$

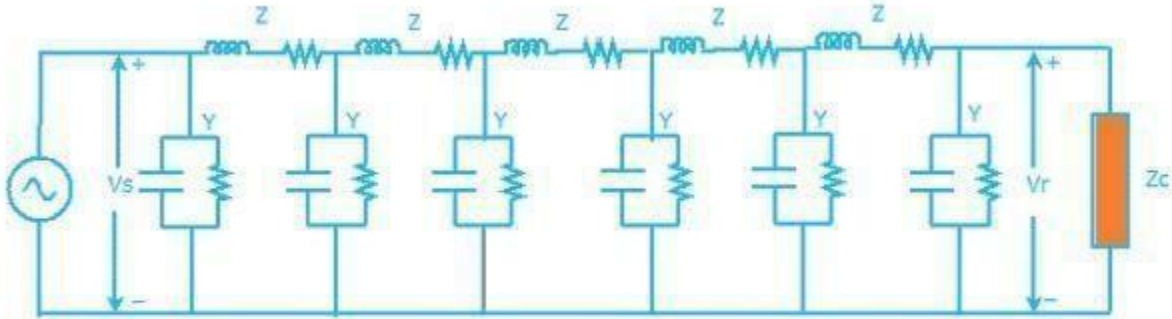
$$I_S = V_R \sqrt{\frac{Y}{Z}} \sinh \sqrt{Y Z} + I_R \cosh \sqrt{Y Z}$$

It is helpful to expand hyperbolic sine and cosine in terms of their power series.

$$\cosh \sqrt{Y Z} = \left(1 + \frac{Z Y}{2} + \frac{Z^2 Y^2}{24} + \dots \right)$$

$$\sinh \sqrt{Y Z} = \left(\sqrt{Y Z} + \frac{(Y Z)^{3/2}}{6} + \dots \right)$$

LONG TRANSMISSION LINE (ABCD PARAMETERS)



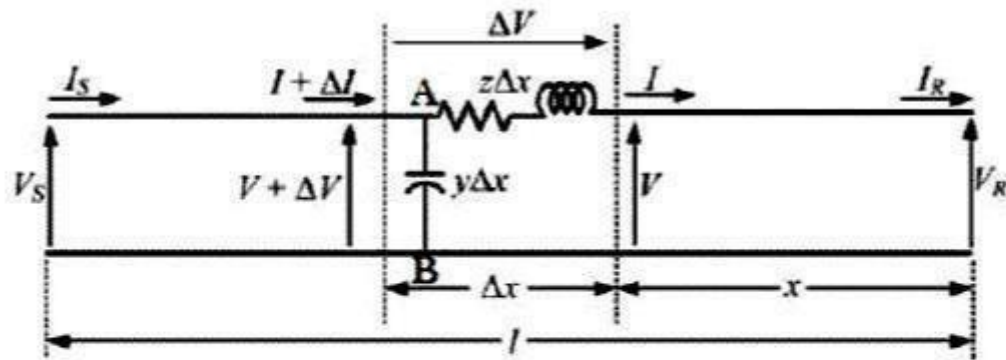
Long Transmission Line model

A power transmission line with its effective length of around 250 Kms or above is referred to as a **long transmission line**. Calculations related to circuit parameters (ABCD parameters) of such a power transmission is not that simple, as was the case for a short or medium transmission line. The reason being that, the effective circuit length in this case is much higher than what it was for the former models(long and medium line) and, thus ruling out the approximations considered there like.

- Ignoring the shunt admittance of the network, like in a small transmission line model.
- Considering the circuit impedance and admittance to be lumped and concentrated at a point as was the case for the medium line model.

Rather, for all practical reasons we should consider the circuit impedance and admittance to be distributed over the entire circuit length as shown in the figure below.

The calculations of circuit parameters for this reason is going to be slightly more rigorous as we will see here. For accurate modeling to determine circuit parameters let us consider the circuit of the **long transmission line** as shown in the diagram below.



Long Transmission Line.

Here a line of length $l > 250\text{km}$ is supplied with a sending end voltage and current of V_S and I_S respectively, where as the V_R and I_R are the values of voltage and current obtained from the receiving end. Lets us now consider an element of infinitely small length Δx at a distance x from the receiving end as shown in the figure where.

V = value of voltage just before entering the element Δx . I = value of current just before entering the element Δx . $V + \Delta V$ = voltage leaving the element Δx .

$I + \Delta I$ = current leaving the element Δx . ΔV = voltage drop across element Δx . $z\Delta x$ = series impedance of element Δx $y\Delta x$ = shunt admittance of element Δx

Where $Z = z l$ and $Y = y l$ are the values of total impedance and admittance of the long transmission line.

$\therefore$ The voltage drop across the infinitely small element Δx is given by $\Delta V = I z \Delta x$

$$\text{Or } I z = \Delta V / \Delta x$$

$$\text{Or } I z = dV / dx \text{ ————— (1)}$$

Now to determine the current ΔI , we apply KCL to node A. $\Delta I = (V + \Delta V)y\Delta x = V y\Delta x + \Delta V y\Delta x$

Since the term $\Delta V \Delta x$ is the product of 2 infinitely small values, we can ignore it for the sake of easier calculation.

$\therefore$ We can write $dI/dx = V y$ —————

—(2) Now differentiating both sides of

eq (1) w.r.t x , $d^2 V/dx^2 = z dI/dx$

Now substituting $dI/dx = V y$ from

equation (2) $d^2 V/dx^2 = zyV$

or $d^2 V/dx^2 - zyV = 0$ —————(3)

The solution of the above second order differential equation is

given by. $V = A_1 e^{x\sqrt{yz}} + A_2 e^{-x\sqrt{yz}}$ —————(4)

Derivating equation (4) w.r.to x .

$dV/dx = \sqrt{(yz)} A_1 e^{x\sqrt{yz}} - \sqrt{(yz)} A_2 e^{-x\sqrt{yz}}$ —————

(5) Now comparing equation (1) with equation (5)

$$I = \frac{dV}{dx} = \frac{z A_1 e^{x\sqrt{yz}}}{\sqrt{(z/y)}} - \frac{z A_2 e^{-x\sqrt{yz}}}{\sqrt{(z/y)}} \text{-----(6)}$$

Now to go further let us define the characteristic impedance Z_c and propagation constant δ of a long transmission line as

$Z_c =$

$\sqrt{(z/y)}$

$) \Omega$

δ

$= \sqrt{(yz)}$

Then the voltage and current equation can be expressed in terms of characteristic impedance and propagation constant as

$V = A_1 e^{\delta x} + A_2 e^{-\delta x}$ —————(7)

$I = A_1 / Z_c e^{\delta x} + A_2 / Z_c e^{-\delta x}$ —————(8)

Now at $x=0$, $V= V_R$ and $I= I_R$. Substituting these conditions to equation (7)

and (8) respectively. $V_R = A_1 + A_2$ —————(9)

$I_R = A_1 / Z_c + A_2 / Z_c$ —————(10)

Solving equation (9)

and (10), We get

values of A_1 and A_2

as,

$$A1 = (VR + ZCIR)$$

$$/2 \text{ And } A1 = (VR$$

$$- ZCIR)$$

Now applying another extreme condition at $x=l$, we have $V = VS$ and $I = IS$.

Now to determine VS and IS we substitute x by l and put the values of $A1$ and $A2$ in equation (7) and (8) we get

$$VS = (VR + ZCIR)e^{\delta l/2} + (VR - ZCIR)e^{-\delta l/2} \text{ ————— (11)}$$

$$IS = (VR/ZC + IR)e^{\delta l/2} - (VR/ZC - IR)e^{-\delta l/2} \text{ ————— (12)}$$

By trigonometric and exponential operators we

$$\text{know } \sinh \delta l = (e^{\delta l} - e^{-\delta l})/2$$

$$\text{And } \cosh \delta l = (e^{\delta l} + e^{-\delta l})/2$$

$\therefore$ equation (11) and (12) can be re-

written as

$$VS = VR \cosh \delta l + ZCIR$$

$$\sinh \delta l IS = (VR \sinh$$

$$\delta l)/ZC + IR \cosh \delta l$$

Thus comparing with the general circuit parameters equation, we get the ABCD parameters of a long transmission line as,

$$C = \sinh \delta l / ZC \quad A = \cosh \delta l \quad D = \cosh \delta l \quad B = ZC \sinh \delta l$$

CIRCLE DIAGRAMS

Transmission line problems often involve manipulations with complex numbers, making the time and effort required for a solution several times greater than that needed for a similar sequence of operations on real numbers. One means of reducing the labor without seriously affecting the accuracy is by using transmission-line charts. Probably the most widely used one is the Smith chart. Basically, this diagram shows curves of constant resistance and constant reactance; these may represent either input impedance or load impedance. The latter, of course, is the input impedance of a zero-length line. An indication of location along the line is also provided, usually in terms of the fraction of a wavelength from a voltage maximum or minimum. Although they are not specifically shown on the chart, the standing-wave ratio and the magnitude and angle of the reflection coefficient are very quickly determined. As a matter of fact, the diagram is constructed within a circle of unit

radius, using polar co-ordinates, the basic relationship upon which the chart is constructed is

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

The impedances which we plot on the chart will be normalized with respect to the characteristic impedance. Let us identify the normalized load impedance as z_L

$$z_L = r + jx = \frac{Z_L}{Z_0} = \frac{R_L + jX_L}{Z_0}$$

$$\Gamma = \frac{z_L - 1}{z_L + 1}$$

$$z_L = \frac{1 + \Gamma}{1 - \Gamma}$$

